Animation of Ptolemy's Cosmos

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More than any other, the study of astronomy contributes to make us better, making us more aware of what is good and beautiful in the moral life. For those who study this subject find a harmony between the Divine things and the beautiful order of the propositions. This makes them love this Divine Beauty and makes them accustomed to take it as a model of their conduct.

Claudius Ptolemy, Almagest [OP 31]

Introduction

Claudius Ptolemy lived in Alexandria, Egypt around 100AD - 170AD. He was a mathematician and astronomer. He developed a geocentric model of the movements of the Sun, Moon and planets and it remained the most accepted theory for over 1200 years.

His best-known publication was entitled *Mathematical Syntaxis* (Μαθηματικὴ Σύνταξις) and later as *The Great Treaties* or simply *The Greatest* (اَلْمَجِسْطِيّ, al-majisti). Through the centuries it became known as "The Almagest".

Ptolemy's objective was to develop a model that would predict the positions of the Sun, Moon and planets at any given time. This was in contrast to earlier philosophers who aimed only at predicting significant events such as eclipses and oppositions. Most books relating the history of astronomy refer to Ptolemy's model, but fail to reveal the beauty and complexity of the work. My objective is to display the details of his model, in an animated form, with as much accuracy as possible.

Two main sources of information for the development of the animations are:

Toomer, G. J., *Ptolemy's Almagest*, Duckworth 1984. [GJT]

Pedersen, O., *A survey of the Almagest (with Annotation and New Commentary by Alexander Jones),* Springer, 2010 [OP]

The Universe

The overall structure of the universe, as seen by Ptolemy, was:

- The heavens (universe) is a sphere carrying the stars.
- The celestial sphere rotates, once per day, with Earth at its centre.
- The Earth is a sphere comprising earth, water, air and fire as described by Aristotle.
- The Earth is insignificantly small compared to the distance of the stars.
- The Earth is at rest, (i.e., does not move, does not rotate).
- The "planets" (Sun, Moon, Mercury, Venus, Mars, Jupiter and Saturn) move within the celestial sphere with uniform circular motion.

Geometric constructs

Ptolemy followed the tradition of earlier Greek philosophers who believed that uniform motion in a circle should be the only allowable motion. He succeeded in using only circles but did not always follow the uniform motion rule.

We now know that the Earth and planets orbit the Sun, and that the Moon orbits the Earth. We also know that planetary orbits are elliptical, not circular. Ptolemy had a difficult job to replicate their apparent movements using circles and uniform motion.

In the models some common constructs are used:

- Earth: The Earth is always in the centre of the universe and is consists of the spheres of Earth, Water, Air and Fire as described by Aristotle.
- Ecliptic Mean Sun: This is the path the Sun would have if it moved with uniform motion round the Earth once per year.
- Epicycle circle: A small circle, the centre of which travels on the circumference of a larger (deferent) circle. The epicycle carries a planet moving around its circumference.
- Deferent circle: A circle around the circumference of which the centre of an epicycle travels, but <u>not</u> with uniform angular motion relative to its centre or as seen from Earth. The centre of a deferent circle is called the "eccentric", and it is placed some distance from Earth.
- Equant circle: A circle from the centre of which the angular daily motion of an epicycle-centre is seen as uniform. The centre of the equant is some distance from Earth but not in the same location as the eccentric.

The important concept is that non-uniform motion as seen from Earth is created by projecting uniform motion from a point some distance from Earth. This is at odds with the original Greek philosophy of perfect circles and motions but was useful in creating a theory that produced positions with acceptable accuracy. The compromise of having some circles not centred on Earth, and some elements of the model not moving with uniform motion, caused much discussion and criticism over the centuries. However, there was no serious challenge to Ptolemy's model until Copernicus produced his heliocentric alternative.

Celestial coordinates

Ptolemy used the ecliptic as his reference plane and the vernal equinox (♈) as the reference point. Celestial positions are described in terms of longitude (relative to ♈) and latitude relative to the ecliptic.

- This is quite different from today's practice where longitude (right ascension) and latitude (declination) are measured relative to the plane of the equator and the "fixed" stars. Ptolemy believed that the "sphere of fixed stars" revolved 1 degree per century around the Earth to produce the effect we now know as precession of the equinoxes.

Longitude is measured eastward along the ecliptic in degrees from the vernal equinox, ♈. A "year" is the tropical year - the time the Sun takes to return to the same longitude on the ecliptic.

Ptolemy treats latitude independently from longitude. In the Almagest he uses a complex "crank" mechanisms to generate latitude. However, in his later work, *Planetary Hypotheses*, he improves the theory of latitudes, and it is this improved version that is used in these animations.

In the animations, the 3D space coordinates of each element (line, circle, planet) is calculated then projected onto the screen according to the user's viewing angle.

- A system of equants, deferents and epicycles is first constructed in the plane of the ecliptic giving $(\mathbf{x}, \mathbf{y})$ spatial coordinates.
- A plane is defined at the required inclination to the ecliptic. The inclined plane intersects the ecliptic along the "node-line". The ascending node is the point where a planet passes from south of the ecliptic to north of the ecliptic. The longitude of the ascending node, $\boldsymbol{\Omega}$, for each planet is determined from the parameters supplied in the Almagest.
- When the radius of a circle is R, the longitude of a point on its circumference is λ, the ascending node of the planet's orbit is Ω, and the inclination is i, then the z spatial coordinate of the point is given by $z = R \sin i . \sin(\lambda - \Omega)$.

This not quite correct in terms of "tilting" the orbit from the ecliptic plane, but the error is small (compared to other errors) and is neglected by Ptolemy.

Parameters

Parameters supplied in The Almagest include the radii of the various circles, the inclination of orbits and the position of the node line, the eccentricity (i.e. distance of the centre of circles from Earth), and the daily rates of angular motion.

Most values are taken from the Almagest [OP 423+] but improved values from Ptolemy's Hypotheses [OP 391] are used where available.

Solex

Solex is a piece of software developed by Aldo Vitagliano. It simulates planetary motion with a high degree of accuracy using JPL Development Ephemeris as a base. It shows the position the planets would have had to within a few arcseconds, throughout the relevant period.

Solex is used to help check the operation of the animated models using the following parameters:

- Topocentric (31.197N 29.89E) to reflect observations from Alexandria. Ptolemy reduced most observations to Alexandria even if they were not made there.
- The reference plane is the ecliptic.
- Equinox of Date is used with Precession, Nutation, Aberration, Light-time and Delta-T all selected to give the best approximation to an observer in Alexandria at that time.
- The "Horizontal" setting gives Local Time (LT) and allows us to recreate the conditions of old observations and to determine their Julian Day number.
- "Reference plane Ecliptic" allows us to view longitude and latitude of planets.
- "Osc. Elements" allows us to check the position of the node line and apogee.

Solex is used to obtain a Julian Day number for observations quoted in the Almagest and to check that the animation is producing planetary positions that are reasonably consistent with reality, considering that they are generated by a ~2000-year-old theory and calibrated using observations up to ~2700 years ago.

Epoch (Radix)

The starting conditions for the model are stated by Ptolemy to be noon Alexandria time on the first day of the reign of King Nabonassar 1st of Babylon. That has been established as BC747 Feb. 26th.

The position of the Sun at the Epoch was based on an observation [GJT 167] of an Autumnal Equinox on "*seventeenth year of Hadrian, on Athyr [III] 7 in the Egyptian calendar, about 2 equinoctial hours after noon*". This observation was on 132AD Sept. 25th 14:00 local time in Alexandria. From Solex, we see that date is J1769539.0. This observation is claimed to be "*one of the most accurate*".

Ptolemy calculated, using a list of kings available to him, that the epoch was 879 Egyptian years (each of 365 days), 66 days and 2 hours before the observation of the equinox. On that basis, the epoch was 320901.0833 days earlier, at J1448637.916667. From Solex we can see that is in fact noon on 747BC Feb 26.

> *Note that I adopt the use of Julian Days rather than the complex set of calendars and kings lists that Ptolemy used.*

The observation described above placed the mean Sun at 116°.66 "to the rear of apogee", [GJT 167], or at longitude 182°15. Ptolemy calculated from the Kings Lists that, at the time of the observation, motion since the epoch was 211°.43 beyond complete revolutions. So, from his model, the position of the mean Sun at epoch was 265°.25 to the rear of the apogee, or ecliptic longitude 330.75. From the table of anomalies, and by calculating the angle Sun-Earth-Mean Sun, we see the true Sun was 2°.39 in advance of the mean at 333.14. The animation starts with the Sun and mean Sun at these positions and reproduces the observation of 132AD to within a small fraction of a degree.

Scale

Ptolemy did not know the relative distances of the various planets and he makes all of the models the same size. The deferent circles usually R=60 partes in radius.

- A "parte" is an arbitrary size and has no physical meaning.

Planetary Hypothesis

Ptolemy later published a document called the *Planetary Hypotheses* which includes some small changes to the parameters and some significant changes to his handling of latitude [OP 391]. In the animations, I have adopted these changes, including the handling of deferent planes (Mercury, Venus), epicycle planes (Mercury, Venus) and placed the plane of the epicycles of the superior planets (Mars, Jupiter, Saturn) parallel to the ecliptic instead of the "crank" mechanism used in the Almagest. These changes make good sense with today's understanding and represent Ptolemy's best model.

Toolbar

The animations are shown with a "toolbar" allowing the user to start/stop, zoom in/out, reverse, tilt, rotate and control the speed of animation. The diagram can be moved by mouse drag.

Development

The animation is written in JavaScript using NetBeans. The scripts are available (with this document) on GitHub. The website is published on the Interplanetary File System (IPFS) using Fleek.

The Animations

> *.. all their apparent anomalies can be represented by uniform circular motions, since these are proper to the nature of divine beings, while disorder and nonuniformity are alien.* [GJT 391]

In the animations, symbols are drawn to represent elements of Ptolemy's model. These can include planets, direction indicators like the vernal equinox or ascending node, and circles like the deferent and epicycle.

The creation of the diagram for each element and planet is described below in the sequence of growing complexity. This has no implications for the actual sequence of the planets.

Earth

The Earth is at the centre of the universe and is the origin of the coordinate system. This is drawn as a set of concentric discs representing Earth, Water, Air and Fire according to Aristotle. The size of the symbols has no significance as Ptolemy did not express views regarding the true size or scale of his models.

Vernal Equinox - ♈

The planes of the ecliptic and equator meet at two points in the sky, the vernal equinox and the autumnal equinox. The vernal equinox is the point where the Sun crosses from south of the equator to north of the equator in springtime (northern hemisphere). This point is sometimes called the First Point of Aries and is represented by the symbol ♈.

Ptolemy regards the direction of ♈ as the direction of longitude zero. All his measurements of longitude are made in degrees from ♈ along the plane of the ecliptic and his measurements of latitude in degrees north or south of the ecliptic.

- The vector (Earth→♈) is the $\boldsymbol{x}$ axis of the coordinate system, $\boldsymbol{y}$ is perpendicular to (Earth→♈) in the plane of the ecliptic and $\boldsymbol{z}$ is perpendicular to the plane of the ecliptic.
- The vector (Earth→♈) is drawn on every diagram and is a fixed line in 3D space.

Mean Sun - λm☉

The mean Sun is the direction of the Sun, as seen from Earth, if it were moving at a uniform angular rate throughout the year. This angular rate or "mean motion" was precisely known by Ptolemy based on observation over several hundred years. The longitude of the mean Sun is represented by λm☉.

- The vector (Earth→λm☉) revolves around Earth with uniform mean motion, starting at longitude 330°.75, (see Epoch above).

Longitude of the Apogee - λa

The longitude of the apogee is the direction where a planet is at its greatest distance from the Earth. It is the direction where an "eccentric" (see below) is to be placed so as to cause variation in the apparent angular motion of a planet in its orbit.

The longitude of the apogee is represented by λa. The Sun has a fixed λa, but for other bodies λa varies according to the motions of each planet.

- The vector (Earth→ λa) is always drawn to show the direction of the apogee.

Longitude of the Ascending Node - Ω

The orbits of the bodies are generally tilted from the plane of the ecliptic, (except the Sun which is, by definition, on the ecliptic). The planes of the ecliptic and orbit intersect along the "node line".

The point where the planet crosses from south to north of the ecliptic is called the "ascending node" and the direction of this point, as seen from Earth, is the longitude of the ascending node, Ω.

- The vector (Earth→Ω) is drawn for each body except the Sun.

The method of calculating the position of the ascending node varies by planet.

Sun

Ptolemy based his theory of the Sun on the work of Hipparchus.

- The period of the Sun is one tropical year. This is the time taken to return to the same ecliptic longitude. Ptolemy used Hipparchus's estimate of the year which is 6 minutes too long due to measurement inaccuracies that are not entirely understood. The 6 minutes has consequences for the accuracy of all his planetary motion models.

The Sun revolves around the Earth once per year. However, its apparent angular velocity varies during the year. We now know this is due to the eccentricity of Earth's orbit round the Sun. This effect can be approximated by having the Sun move at uniform speed round a circle, called the "deferent", with its centre, called the "eccentric", some distance from the centre of the Earth.

- To fit the observations, the centre of the deferent, D, must be placed 1/24th of the radius of the deferent circle in the direction of λa = 65°.5. This value is fixed in Ptolemy's model although we now know it moves very slowly due to presession of Earth's orbit.
- The deferent circle is drawn with radius 60p and the eccentric at D is 2.5p from the Earth in the direction of λa. ("p" is a "parte" and has no physical meaning).
- The Sun travels round the deferent circle with uniform motion (i.e., the same rate as the mean Sun) to complete the orbit in one tropical year. The motion is uniform as seen from D, but it varies as seen from Earth in a manner that is fairly consistent with observations.

The initial value (called radix) of the longitude of the mean Sun was obtained as explained in the section "Epoch" above. The animation is consistent the observation [OP Obs. 62] made by Ptolemy on 132 AD Sept. 25th 14:00 which he used to determine the date of the epoch.

In the animation:

- The fixed position of λa is drawn on the outer circumference and it is joined to Earth.
- The deferent and its centre are drawn, with centre D spaced 2.5p in the direction of λa from Earth.
- The angle of the Sun around the deferent is calculated from its longitude at epoch and mean daily motion. The 3D space coordinate of the Sun on the deferent is calculated. Note that the longitude of the vector (E→Sun) at the epoch is 2°.39 more than that of the mean Sun as can be found by solving the triangle Earth-D-Sun.
- A line is drawn from D to the position of the Sun.

The body of the Sun is drawn and joined to Earth, and the bearing of the vector (Earth→Sun) that determines the longitude of the true Sun.

Superior Planets

The challenge with Superior planets (Mars, Jupiter, Saturn) is to replicate their apparent periods of retrograde motion as well as their variations in apparent angular velocity.

- We now know that the apparent retrograde motion of these planets is due to the fact that they orbit the Sun rather than the Earth. These planets appear to move backwards simply because the Earth is "overtaking" them in its own orbit round the Sun. Ptolemy uses a circle-on-circle approach called an "epicycle", to achieve this motion.

- Variation in apparent angular velocity we know to be due to the orbits being elliptical rather than circular. As with the Sun, this effect can be (partially) achieved by using a deferent circle with its centre offset from Earth. However, motion of the centre of the epicycle around the deferent is not uniform. Therefore, another construct, called the "equant", is introduced as a point from which motion round the deferent does appear uniform.

Each of the superior planets has two important periods:

- The mean tropical period – return to the same longitude around the ecliptic. This provides the mean motion of movement round the equant - Mars 1.88 tropical years, Jupiter 11.86 tropical years, Saturn 29.49 tropical years.
- The mean synodic period, from opposition to opposition (called anomalistic period by Ptolemy), Mars 779.94 days, Jupiter 398.88 days, Saturn 378.09 days. This provides the mean motion of movement round the epicycle.

Ptolemy must have had access to a long history of observations because his estimates of these periods is remarkably accurate.

The longitude of the apogee, λa, for superior planets, was believed to be fixed relative to the stars, and the stars were believed to revolve round Earth by 1 degree per century, west-to-east, relative to the vernal equinox.

In the animation:

- The positions of the apogee, λa, and node, Ω, are calculated from their longitude at epoch and the very slow motion of the stars. They are drawn on the outer circumference and joined to Earth.
- An equant circle of radius R is drawn with centre E, distant 2e from the Earth, in the direction λa.
- The point S is drawn travelling round the equant circle with uniform motion, west-to-east, with a period equal to the mean tropical period of the planet.
- A deferent circle of radius R is drawn with centre D, distant 1e from Earth, in the direction of λa.
- The point C travels round the Equant circle such that its motion, as seen from E is uniform, i.e., the vector (D→C) passes through S.

The combination of equant and deferent circles seeks to replicate the variations in apparent angular motion seen during a complete period of the planet. The technique of using D and E spaced 1e and 2e from Earth is known as "Bisection of Eccentricity". It is introduced in the Almagest, but Ptolemy does not explain how he decided on this construct.

- An epicycle is drawn with centre C and radius r. The planet moves round the epicycle, west-to-east, at a uniform rate such that it takes one synodic (anomalistic) period to complete the circle. Inspection of the geometry concludes that the planet moves such that the vector (C→Planet) is always parallel to the vector (Earth→mean Sun).
- Motion round the epicycle is west-to-east so that the planet is nearest Earth, (apparently brighter), at opposition.

The epicycle seeks to replicate the apparent retrograde motion of the planet for a period before and after opposition. The size of the epicycle is designed to show the length and duration of the retrograde path.

To produce a latitude measurement the equant and deferent circles are projected onto a plane set at the "inclination" angle of the planet's orbit. However, as specified in the Hypotheses, the plane of the epicycle remains parallel to the ecliptic.

Mars

Basic parameters are taken from [OP 427]. R=60p, e=6.0p, r=39.5. Inclination 1°.83 is from the Hypotheses [OP 392].

The observation [OP Obs. 70]: 135AD Feb 21st 21:00 Alexandria time, *longitude at opposition 148°83*, is reproduced to within 0°.06. Latitude is not quoted in the observation but the latitude in the animation is 0°.16 fom the Solex value.

Jupiter

Basic parameters from [OP 428]. R=60, e=2.75, r=11.5. Inclination is 1°.5 is from the Hypotheses [OP 392].

The animation was tested using the observation [OP Obs. 75]: 136AD Aug 8th 22:00 Alexandria time, *longitude at opposition 337°.9*. However, the planet is not at opposition in the animation.

SOLEX shows that opposition was actually on 3rd Sept. around 9:00 local time. The SOLEX and Almagest positions are reproduced by the animation correctly to within a fraction of a degree using that alternative date. It is presumed that the date must have been wrongly recorded or translated at some point in the past. The corrected date is used in the animation.

Saturn

Basic parameters from [OP 429]. R=60, e=3.4155, r=6.4155. Inclination 2°.5 from the Hypotheses [OP 392].

The observation [OP Obs. 55]: 127AD Mar 26 evening Alexandria time, *opposition of Saturn at longitude 181°.217* is reproduced to within a fraction of a degree in longitude and latitude.

Moon

The Moon is important because it is the only genuine geocentric "planet". It is also close enough that detailed observations can be made. The Moon is much more irregular than the Sun or the Superior Planets and several different periods can be considered:

- Tropical month – return to same point on the ecliptic.
- Synodic month – period full moon to full moon.
- Draconitic month – return to same latitude.
- Anomalistic month – return to the same angular velocity.
- Sidereal month – return to the same longitude relative to the stars, (not used by Ptolemy).

Ptolemy used fifteen lunar eclipses, between 721 BC to 136AD, to estimate these periods and invented mathematical methods to reduce observations to the parameters of his model. All of these periods vary from month to month and only average (mean) values can be established.

In his initial model, Ptolemy used an epicycle to create the variations in angular motion that we now know to be caused by the Moon's elliptical orbit. He found that an epicycle, with its centre moving with uniform motion around a circle centred on Earth, would give good predictions for new and full Moon but was poor at first and last quarters (quadrature). We now know this is caused by the Sun's gravity and is called "evection". Ptolemy was the first to recognise its effects and modified the model accordingly:

- The epicycle was changed such that its centre travels around a deferent circle with its centre (the eccentric) some distance from Earth.
- In addition, the longitude of the apogee, λa, (i.e., the direction of the eccentric as seen from Earth) is made to revolve around the Earth in a retrograde motion with a synodic period. In effect, this brings the centre of the epicycle closer to Earth during quadrature, increasing its apparent diameter and improving the match with observations.

There were still errors at other times of the month (near the octants), so Ptolemy made one more change:

- A new point was defined, opposite the eccentric, called the "prosneusis" and denoted by D' in the diagram.
- The motion of the Moon round its epicycle was determined to be uniform angular velocity relative to the vector (D'→C). This has the effect of giving the Moon a little "advance" or "retard" and improves the match with observations at the octants.

The latitude of the Moon is established by first calculating all the elements of the model in the ecliptic plane then projecting them onto a plane tilted by 5° to the ecliptic along the node line Ω. The node line moves in a retrograde motion by an amount equal to the mean draconic period minus the mean tropical period.

Overall, the model works moderately well except for the obvious fact that the distance Earth-Moon varies considerably and is inconsistent with its actual variation in apparent size.

In the animation:

- The mean Moon, λm☾, is drawn revolving around Earth with the uniform angular motion of the synodic period relative to the mean Sun. (i.e., new Moon to new Moon.)
- The longitude of the apogee, λa, is drawn revolving <u>retrograde</u> with the uniform motion of the synodic period relative to the mean Sun. (I.e., λm⊙ bisects the angle between λm☾ and λa.)
- The longitude of the ascending node, Ω, is calculated and drawn rotating slowly at a rate equal to the draconitic minus tropical periods.
- The eccentric centre of a deferent circle, D, is set distance e=10.316p in the direction λa.
- A deferent circle is drawn with radius R=49.6833p, centred on D. This circle carries the epicycle centre C. Because λa is revolving around Earth, the centre of the epicycle moves closer to, and further away from Earth twice per month.
- The centre of an epicycle, C, travels around the deferent circle such that the vector (Earth→C) rotates with uniform angular motion, west-to-east, equal to the Moon's mean synodic period relative to the direction of the mean Sun λm⊙. (i.e., C is in the direction of the mean Moon as seen from Earth.)
- An epicycle of radius r = 5.25p is centred on C.
- A point D' is established in a position diametrically opposite D, and a vector is drawn (D'→C).
- The Moon travels round the epicycle east-to-west at a uniform rate equal to its mean anomalistic period relative to the direction of the vector (D'→C).

To obtain latitude, all elements described above are calculated and projected onto a plane tilted at 5° to the ecliptic along the node line. This is not a correct interpretation of an inclined orbit, but Ptolemy considered the error to be insignificant.

The parameters for the animation are taken from [OP 424]. R=49.683p, e=10.316p, r=5.25p.

Ptolemy's observation on 139AD Feb 9th [OP Obs. 82] is reproducs to within 0°1 in longitude. Latitude is not quoted in the observation but that shown by the animation (4°.73 north) is not far from the Solex value of 4°.66 north.

The observation of a lunar eclipse in Babylon [OP Obs. 11] on BC382 Dec 12th is also reproduced with reasonable accuracy.

Inferior Planets

The special features of Venus and Mercury are that they never appear at opposition to the Sun. They oscillate to the east and west of the Sun, appearing as evening or morning "stars".

Their periods are:

- Tropical periods – the average period of their return to the same position on the ecliptic.
- Synodic (Ptolemy calls it anomalistic) period: the average time between eastern and western elongations.

As usual, their periods are not fixed but we can establish average (mean) values. These planets may be east or west of the Sun but their average position over the long term is the same as the Sun so their mean motion in longitude (tropical period) is the same as the mean Sun. This lead some early astronomers to suspect that Mercury and Venus orbit the Sun (which in turn revolves around Earth). However, Ptolemy does not consider this possibility and he continues to use his constructs of deferents and epicycles.

For the inferior planets:

- The longitude of apogee, λa, is fixed relative to the stars and therefore moves at 1 degree per century.
- The nodes of inclination, Ω, are also fixed relative to the stars and set at 90° behind λa.

Ptolemy had a problem deciding on the longitudes of λa and Ω. His values "at epoch" for these parameter are not very accurate. The value for Ω is very questionable as descibed for the individual planets below.

Both planets have an equant (providing the uniform motion) and a deferent circle carrying the centre of an epicycle centre moving with non-uniform motion.

- Venus has low eccentricty. Its eccentric and equant centre are close to Earth and are fixed along the line to λa.
- Mercury has high eccentricity and its deferent centre is further from Earth and travels in a small circle around Earth, (similar but not identical to the Moon).

Venus

Venus is relatively simple. With low eccentricty:

- The direction of λa and Ω are shown. They are both fixed relative to the stars.
- An equant circle is drawn, centred at E which is located 2e from Earth in direction λa. The point S moves round the equant with uniform motion equal to that of the mean Sun.
- A deferent circle is drawn radius R, centred at D, distant e from Earth in direction λa. Here we have bisection of eccentricty as for the superior planets.
- The centre of an epicycle, C, travels aound the deferent circle such that its motion is uniform as seen from E. C is at the point where the vector (E→S) intersects the deferent circle.
- An epicycle of radius r, centred on C, is drawn and Venus travels around the epicycle with mean anomalistic motion, relative to the vector (E→C).
- Using Ptolemy's theory from the Hypotheses, the deferent has an inclination of 0°.166 but the epicycle has an additional inclination of 3°.5 relative to the deferent. The ascending node of the inclination is the same as for the deferent and epicycle and fixed relative to the stars.

Parameters are taken from [OP 426]. R=60p, e=1.25p, r=43.166p. Inclination, from the Hypotheses [OP 392] is 0°.1666 for the deferent and an additional 3°.5 for the epicycle.

As such, the model fails to produce correct latitudes unless the longitude of Ω is changed from being 90° "behind" λa to being 90° in front of λa. With this value of Ω, the model correctly reproduces observation [OP Obs 66] on 134AD Feb 18th "morning" - *position of Venus relative to a Scorpii giving a maximum western elongation of 43°;35 at a longitude at 281 °;55 with Venus at western elongation*. This corrected value of Ω is used in the animation.

Mercury

Mercury is the most complex problem. We now know it has a relatively high eccentricity (0.2) compared to other planets. It also has a relative short period (87 days) and high inclination (7°). As viewed from Earth, its movements seem quite complicated. With its proximity to the Sun, it is difficult to observe and some of the old observations, used by Ptolemy to calibrate his model, are inaccurate. An epicycle is used to approximate the effects of an elliptical orbit, but a circlular epicycle cannot achieve this in a satifactory way.

- Ptolemy found it difficult to establish the longitude of the apogee, λa, but he eventually settled on a value of 181°.166, fixed relative to the stars. There was some observational evidence supporting this.
- Further confusion occurred when observations (some erroneous) seemed to imply that Mercury has two perigees, one 70° before λa and the other 120° after λa. In order to construct a model approximating these movements, Ptolemy made the centre of the deferent move around a circle (in a manner similar but not identical to the Moon). The centre of the deferent D travels around a small circle with radius 3.0p and centre point F located 6.0p from Earth along the line to λa.
- Looking for a point from which motion can be considered uniform, Ptolemy put the centre of an equant, E, at a distance 3.0p from Earth along the line to λa. The centre of an epicycle travels around the deferent circle such that its motion is uniform as seen from E.
- Finally the planet Mercury travels around the epicycle in a retrograde direction with uniform motion relative to the vector (E→C).

- Using Ptolemy's theory from the Hypotheses, the deferent has an inclination of 0°.166 and the epicycle has an additional inclination of 6°.5 relative to the deferent. The ascending node of the inclination is 90° behind λa and fixed relative to the stars.

Parameters are taken from [OP 425]. R=60p, e=3.0p, r=22.5p. Inclination from the Hypotheses [OP 392] is 0°.1666 for the deferent and an additional 6°.5 for the epicycle.

The model gives accurate longitude when reproducing the observation [OP Obs. 71] on 135AD 5th April "evening" - *Position of Mercury relative to a Tauri giving a maximum eastern elongation of 23°;15 at a longitude of 34°;20*. However, longitude at Epoch is about 9° different from that given by SOLEX.

Latitude at the epoch and the 135AD observation are not accurate. Latitude measurements can be improved greatly by setting Ω at the epoch equal to λa – 160° rather than λa – 90° as suggested in the Almagest. However, the animation runs with Ptolemy's original parameter.

Conclusion

The animation shows the workings of Ptolemy's model and reproduces the observations made during his lifetime with fair accuracy. The diagrams show the constructs described by Ptolemy but there is further work needed to understand how well the model correctly reproduces the real positions of the planets and the many observations mentioned in the Almagest.   In particular the handling of latitude for Mercury and Venus does not appear to opperate correctly when the Almagest parameters are used for λa and Ω.

The website showing these animations can be found at https://mycosmostuff.space.

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